D PROBLEM SOLVING USING PYTHAGORAS

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Step 2: Mark known lengths and right angles on the diagram.
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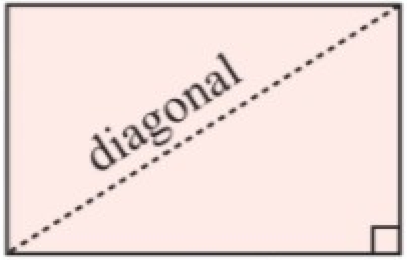
The following special figures contain right angled triangles:



rectangle

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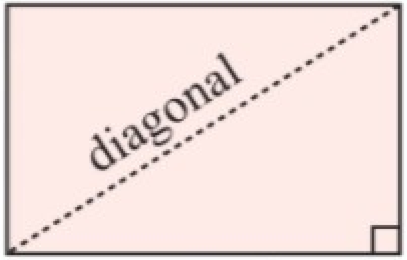

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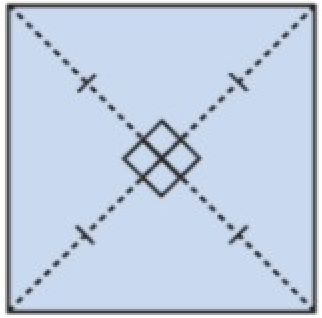
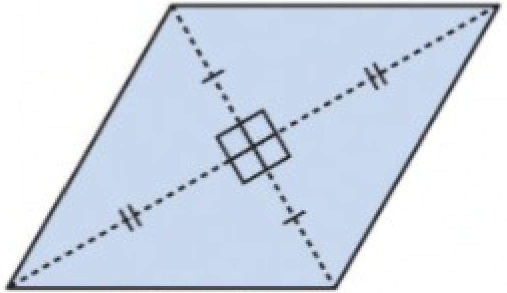


square



rhombus

In a **square** and a **rhombus**, the diagonals bisect each other at right angles.

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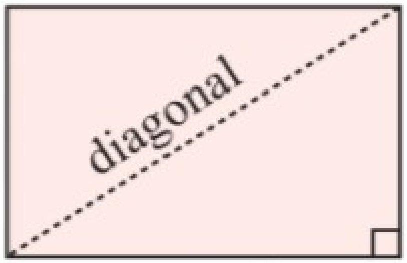

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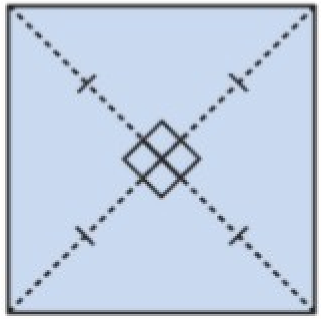

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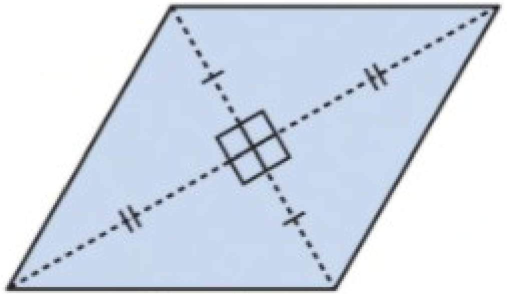

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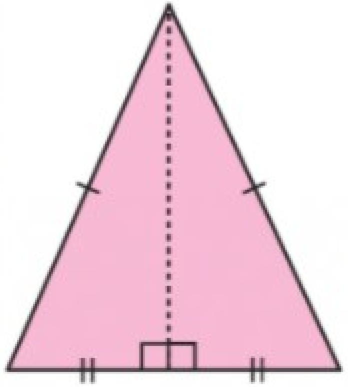
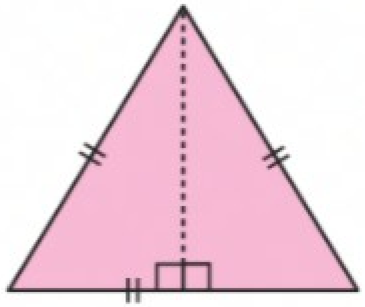


isosceles triangle



equilateral triangle

In an **isosceles triangle** and an **equilateral triangle**, the altitude bisects the base at right angles.

Example 8

Self Tutor

A rhombus has diagonals of length 6 cm and 8 cm.
Find the length of its sides.

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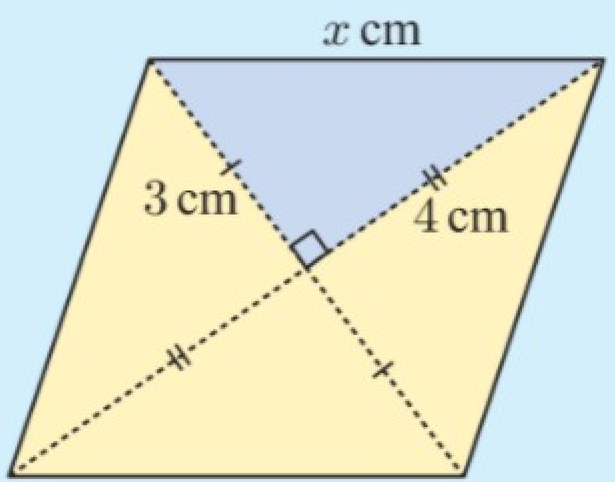


The diagonals of a rhombus bisect at right angles.

Let each side of the rhombus have length x cm.

$$\therefore \quad x^2 = 3^2 + 4^2 \qquad \{\text{Pythagoras}\}$$
$$\therefore \quad x^2 = 25$$
$$\therefore \quad x = 5 \qquad \{\text{as } x > 0\}$$

The sides are 5 cm long.

EXERCISE 5D

1 A rectangle has sides of length 8 cm and 3 cm. Find the length of its diagonals.

2 The longer side of a rectangle is three times the length of the shorter side. The length of the diagonal is 10 cm. Find the dimensions of the rectangle.

3 A rectangle with diagonals of length 20 cm has sides in the ratio $2:1$. Find the:

a perimeter

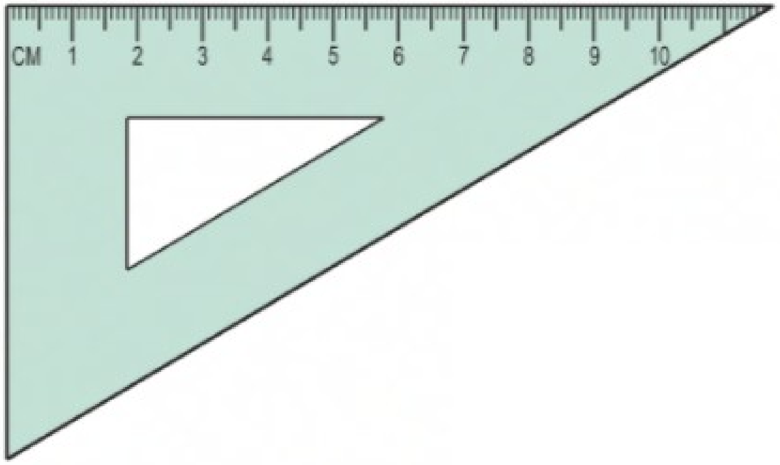
b area of the rectangle.

4 A rhombus has sides of length 6 cm. One of its diagonals is 10 cm long. Find the length of the other diagonal.

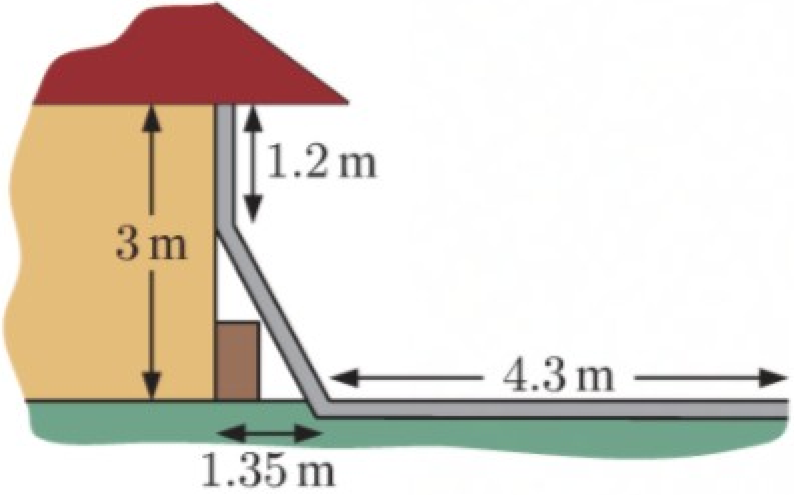
5 A square has diagonals of length 10 cm. Find the length of its sides.

6 A rhombus has diagonals of length 8 cm and 10 cm. Find its perimeter.

7 To check that his set square was right angled, Roger measured its sides. The two shorter sides were 8 cm and 11.55 cm long, and the longest side was 14.05 cm long. Is the set square right angled?



8



A drain pipe runs down the wall of a house, then out to the road as shown. Find the length of the pipe.

Example 9

Self Tutor

A man and his son both leave point A at the same time. The man rides his bicycle due east at 16 $\text{km}\,\text{h}^{-1}$. The son rides his bicycle due south at 20 $\text{km}\,\text{h}^{-1}$. How far apart are they after 4 hours?

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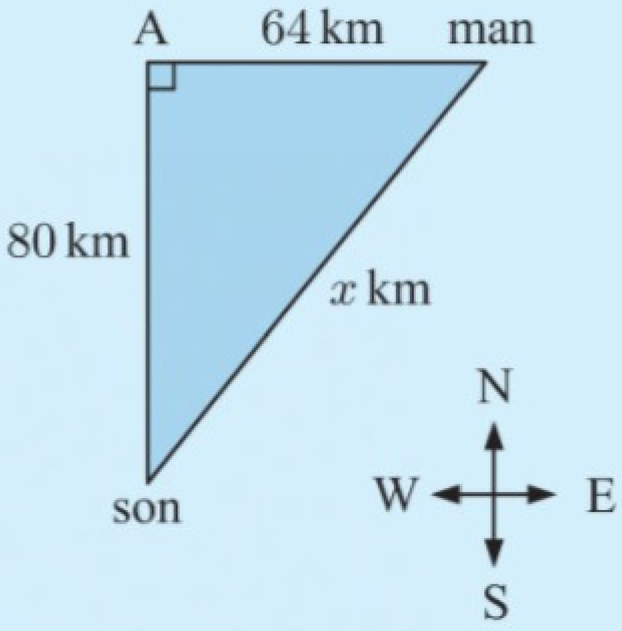

After 4 hours, the man has travelled $4 \times 16 = 64$ km, and his son has travelled $4 \times 20 = 80$ km.

Let the distance between them be x km.

Thus $x^2 = 64^2 + 80^2$ {Pythagoras}

$\therefore \ x^2 = 10\,496$

$\therefore \ x = \sqrt{10\,496}$ {as $x > 0$}

$\therefore \ x \approx 102$

$\therefore$ they are about 102 km apart after 4 hours.

9 A yacht sails 5 km due west and then 8 km due south. How far is it from its starting point?

10 Pirate Captain William Hawk left his hat on Treasure Island. He sailed 18 km northeast through the Forbidden Strait, then 11 km southeast to his home before realising it was missing. He sent his parrot to fetch the hat and return it to the boat. How far did the parrot need to fly?

11 Two runners set off from town A at the same time. One runs due east to town B, and the other runs due south to town C at twice the speed of the first. They both arrive at their destinations two hours later. Given that B and C are 50 km apart, find the average speed of each runner.

13 A highway runs east-west between two towns B and C that are 25 km apart. Town A lies 15 km directly north of B. A straight road is built from A to meet the highway at D. Given that D is equidistant from A and C, find the position of D on the highway.

Example 10

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An equilateral triangle has sides of length 6 cm. Find its area.

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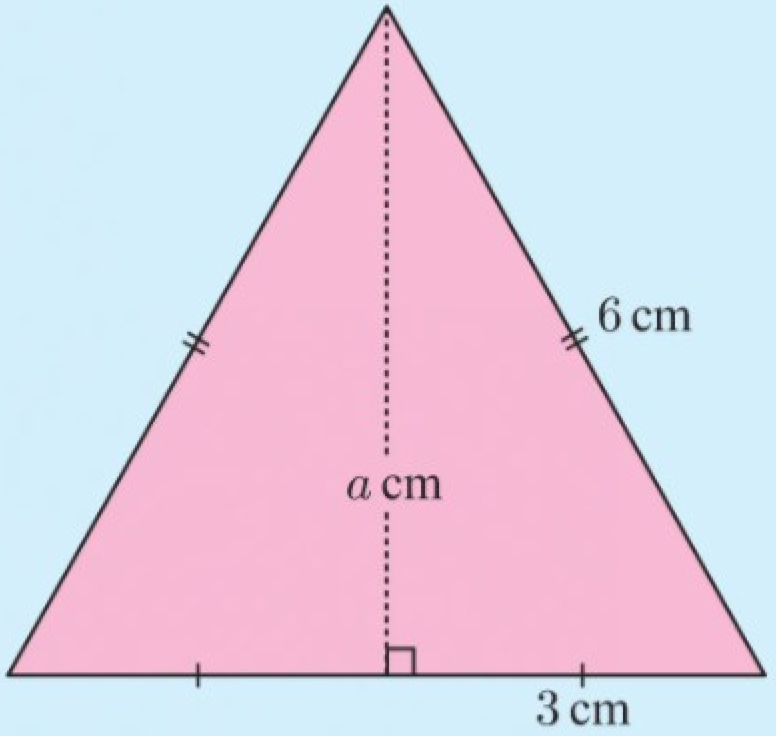
An equilateral triangle has sides of length 6 cm. Find its area.

The altitude bisects the base at right angles.

$$\begin{aligned} \therefore\ a^2 + 3^2 &= 6^2 \qquad \{\text{Pythagoras}\} \\ \therefore\ a^2 + 9 &= 36 \\ \therefore\ a^2 &= 27 \\ \therefore\ a &= \sqrt{27} \qquad \{\text{as } a > 0\} \end{aligned}$$

$$\begin{aligned} \text{Area} &= \tfrac{1}{2} \times \text{base} \times \text{height} \\ &= \tfrac{1}{2} \times 6 \times \sqrt{27} \\ &= 3\sqrt{27}\ \text{cm}^2 \\ &\approx 15.6\ \text{cm}^2 \end{aligned}$$



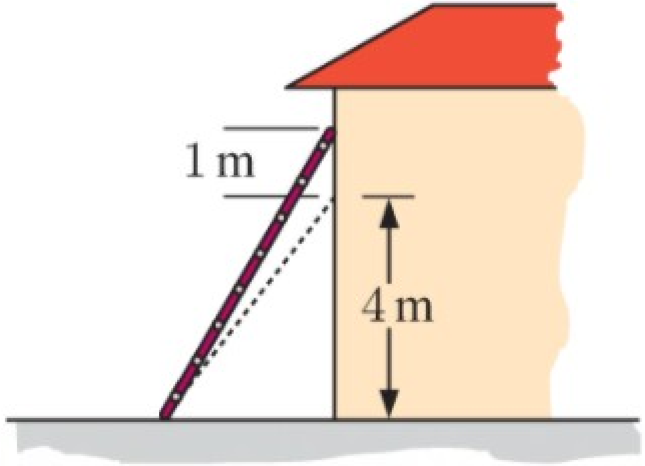
So, the area is about 15.6 cm^2.

14 An equilateral triangle has sides of length 12 cm. Find the length of one of its altitudes.

15 An isosceles triangle has equal sides of length 8 cm and a base of length 6 cm. Find the area of the triangle.

16 An equilateral triangle has area $16\sqrt{3}$ cm^2. Find the length of its sides.

17 An extension ladder rests 4 m up a wall. If the ladder is extended a further 0.8 m without moving its feet, then it will now rest 1 m further up the wall. How long is the extended ladder?



18 A rectangular piece of paper, 10 cm by 25 cm, is folded so that a pair of diagonally opposite corners coincide. Find the length of the crease.

EXERCISE 5D

1 $\sqrt{73}$ cm ≈ 8.54 cm

2 $3\sqrt{10}$ cm by $\sqrt{10}$ cm, or, ≈ 9.49 cm by 3.16 cm

3 **a** $24\sqrt{5}$ cm ≈ 53.7 cm **b** 160 cm^2

4 $2\sqrt{11}$ cm ≈ 6.63 cm **5** $5\sqrt{2}$ cm ≈ 7.07 cm

6 $4\sqrt{41}$ cm ≈ 25.6 cm

7 $8^2 + 11.55^2 = 197.4025$ and $14.05^2 = 197.4025$
$\therefore$ is right angled.

8 7.75 m **9** $\sqrt{89}$ km ≈ 9.43 km

10 $2\sqrt{445}$ km ≈ 42.2 km **11** ≈ 11.2 km/h and ≈ 22.4 km/h

12 **a** ≈ 115.4 m **b** 100 m **c** ≈ 15.4 m

13 D is 8 km from B.

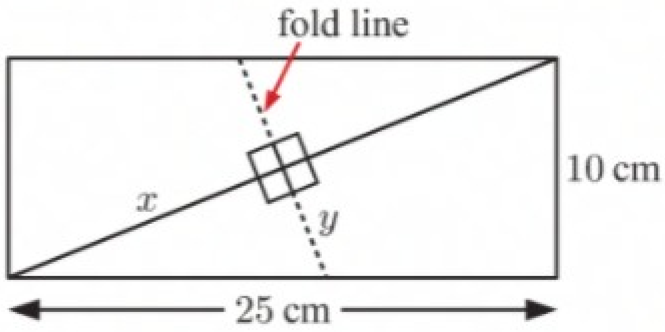
14 $6\sqrt{3}$ cm ≈ 10.4 cm

15 ≈ 22.2 cm^2

16 8 cm

17 6.025 m

18



$x \approx 13.463$, crease ≈ 10.8 cm